

## 3-D Time/Frequency-Range-Doppler Signatures for SAR Imaging of Ground Moving Targets

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### ABSTRACT

*In this paper, we introduce the concept of 3-D radar signatures in a slow time-range-Doppler, fast time-range-Doppler and frequency-range-Doppler. Then, we describe how to utilize 3-D time or frequency-range-Doppler signatures to extract information for detecting, relocating and re-focusing moving targets. Finally, we use AN/APY-6 X-band radar data for the demonstration of ground moving target detection.*

### 1 INTRODUCTION

The returned signal from an object to be imaged can be represented as the integration of the contributions from all scatterers in the object [1]:

$$s_R(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x, y, z) \exp\left\{-j2\pi f_0 \frac{2R_P(t)}{c}\right\} dx dy dz \quad (1)$$

*for*  $2R_P(t)/c \leq t \leq T_{PRI} + 2R_P(t)/c$

where  $\rho(x, y, z)$  is the reflectivity function of a point scatterer  $P$  at  $(x, y, z)$ ,  $f_0$  is the carrier frequency,  $c$  is the speed of electromagnetic wave propagation,  $R_P(t)$  is the distance from the radar to the point scatterer  $P$ , and  $T_{PRI}$  is the pulse repetition interval of the transmitted signal.

After transmitting a sequence of  $N$  pulses, the received baseband I/Q signals are organized into  $N$  pulses and  $M$  range-cells. Thus, a 2-D complex I/Q data array  $s_R(r_m, t_n)$  can be obtained, where  $m = 0, 1, \dots, M-1$ ;  $n = 0, 1, \dots, N-1$ . The  $M$  range-cells are represented in the time-domain, also called the fast time. At each range cell, the data across the  $N$  pulses constitutes a time history series, also called the slow time.

After range tracking and Doppler tracking, the range aligned profiles become  $G(R_m, t_n)$ , ( $m = 0, 1, \dots, M-1$ ;  $n = 0, 1, \dots, N-1$ ). The two-dimensional range profiles are expressed in a range and slow-time domain.

The conventional image formation takes the fast Fourier transform (FFT) for the new time history series and generates an  $N$ -point Doppler spectrum called the Doppler profile. By combining the  $N$ -point Doppler spectrum at each range cells for all  $M$  range profiles, the  $M$ -by- $N$  image is formed

$$I(R_m, f_n) = FFT_{t_n} \{G(R_m, t_n)\} \quad (2)$$

where  $FFT_{t_n}$  denotes the FFT operation with respect to the variable  $t_n$ . Therefore the radar image  $I(R_m, f_n)$  is a target's 3-D reflectivity mapped onto a two dimensional range-Doppler or range and cross-range plane.

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All information about the object is contained in the two-dimensional complex array  $s_R(r_m, t_n)$ , called the fast time and slow time domains. To further exploit the information, the generation of a three-dimensional (3-D) feature space is useful.

**2 3-D FEATURE SPACE**

A 3-D feature space can be generated by using a 2-D time-frequency transform [2] on one of the data dimensions. In the case of SAR imaging, the time-frequency transform can be applied directly to the slow time or the fast time domain, or applied to the Fourier transform of the slow time or fast time domains.

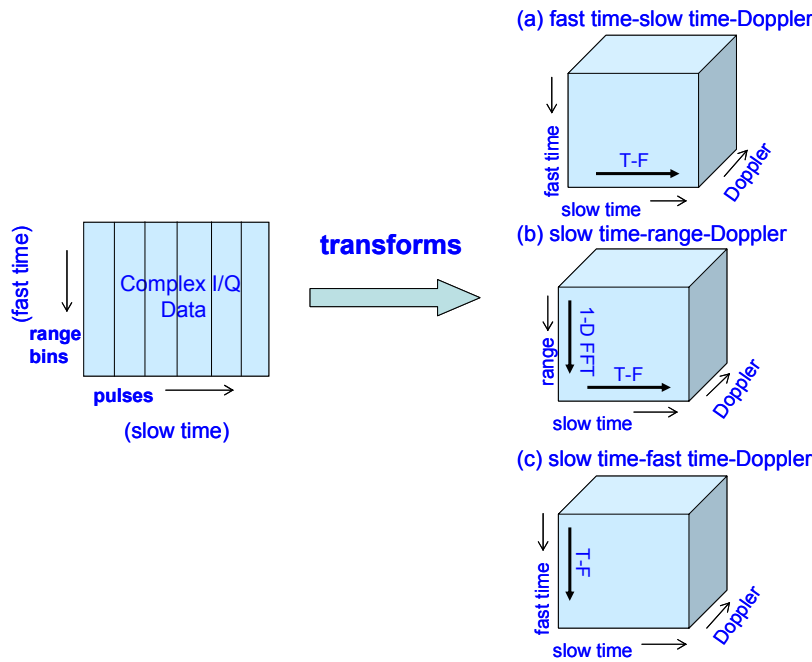


Figure 1. Approaches to generate 3-D feature space.

Figure 1 illustrates some general approaches to generating a 3-D feature space. From a 2-D data space in the fast time and the slow time domains, a 2-D time-frequency transform is necessary to generate a 3-D feature representation. The 3-D feature space can be (a) a fast time-slow time-Doppler space by applying a time-frequency transform to the slow-time domain, (b) a slow time-range-Doppler space by first applying a 1-D FFT to the fast time domain, then applying a time-frequency transform to the slow time domain, and (c) a slow time-fast time-Doppler space by applying a time-frequency transform to the fast time domain, or other combinations.

### 3 SLOW TIME-RANGE-DOPPLER SPACE

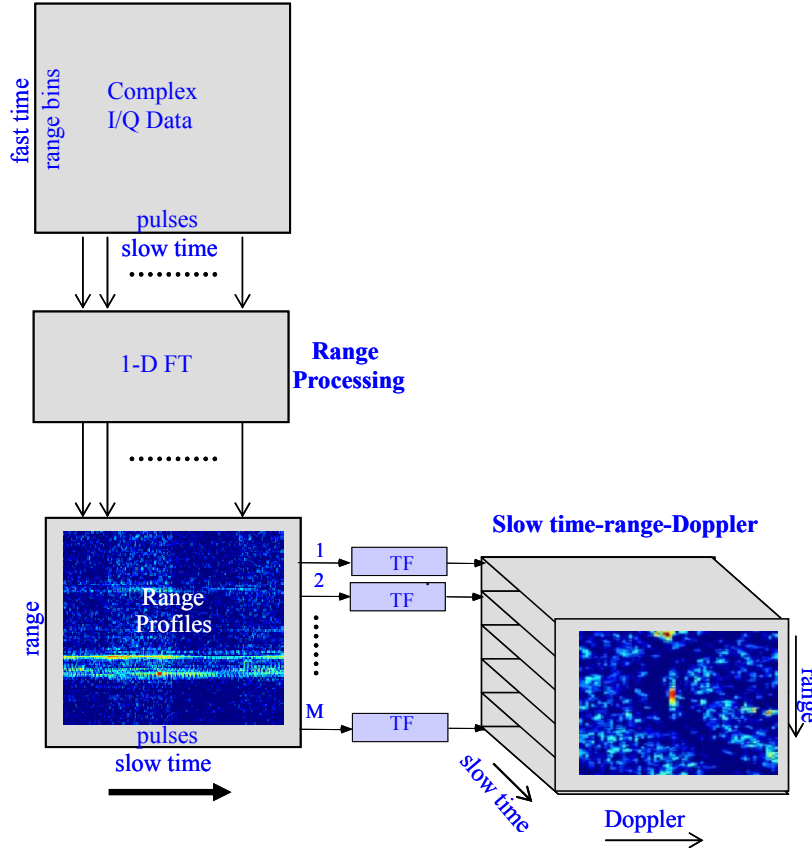


Figure 2. Generate a slow time-range-Doppler feature space.

When the data is formed as a complex 2-D array  $G(R_m, t_n)$ , ( $m = 0, 1, \dots, M-1$ ;  $n = 0, 1, \dots, N-1$ ) with  $M$  slow time history series, each having the length of  $N$  pulses, the 3-D space processing takes a time-frequency transform for each slow time history series and generates an  $N \times N$  slow time-Doppler distribution. By combining the  $N \times N$  time-Doppler distributions for all  $M$  range cells, the  $N \times M \times N$  slow time-range-Doppler cube  $Q_1(t_n, R_m, f_n)$  can be formed:

$$Q_1(t_n, R_m, f_n) = TF_n \{G(R_m, t_n)\}, \quad (3)$$

where  $TF_n$  denotes the time-frequency transform with respect to the variable  $n$ . At each sampling time  $t_i$  ( $i=0, 1, 2, \dots, N-1$ ), only one range-Doppler image frame  $Q_i(R_m, f_n, t_n=t_i)$  can be extracted from the  $N \times M \times N$  slow time-range-Doppler cube. According to the frequency marginal condition, the slow time-range-Doppler feature space and the 2-D range-Doppler image space is related by [1]

$$I(R_m, f_n) = \sum_{t_n=t_0}^{t_{N-1}} Q_1(t_n, R_m, f_n), \quad (4)$$

which means that the 2-D range-Doppler image is just a 2-D slice of the 3-D slow time-range-Doppler space.

Figure 2 shows the typical block diagram of the slow time-range-Doppler processing to generate a 3-D feature space. In short, to retrieve the range information, the FFT is applied to the fast time domain. The complex I/Q 2-D data array becomes a 2-D (slow time and range) array of range profiles. By taking the time-frequency transform in the slow time domain, the slow time-range-Doppler feature space is generated.

#### 4 FREQUENCY-RANGE-DOPPLER SPACE

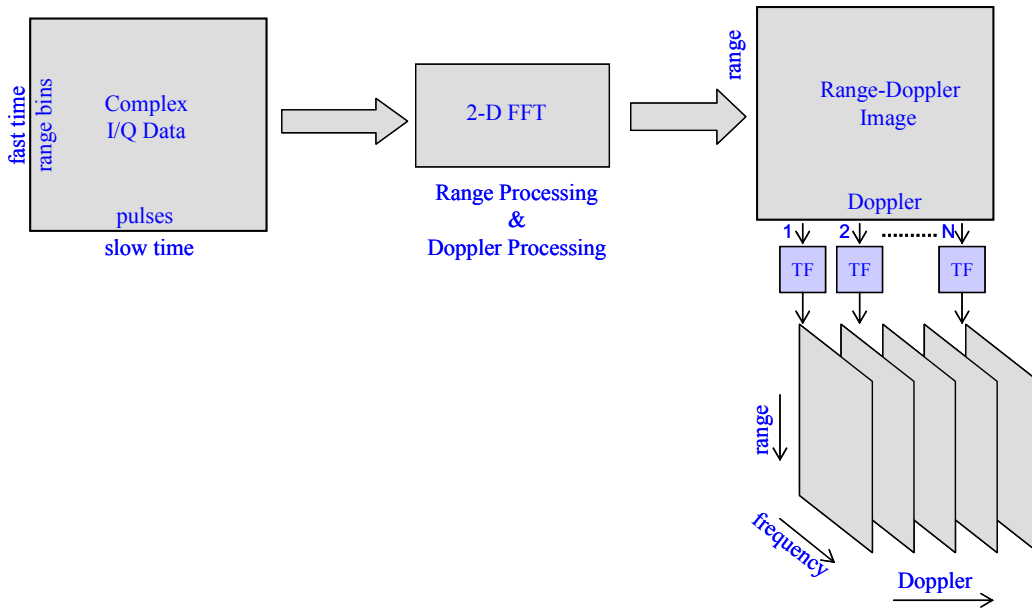


Figure 3. Generate a frequency-range-Doppler feature space.

The 2-D complex I/Q data array  $s_R(r_m, t_n)$ , ( $m = 0, 1, \dots, M-1$ ;  $n = 0, 1, \dots, N-1$ ) in the fast time and slow time domain is used to generate a range-Doppler feature image by applying a 2-D FFT:

$$I(R_m, f_n) = FFT_2\{s_R(r_m, t_n)\} \quad (5)$$

Then, a time-frequency transform is applied to each range profile generating an  $M \times M$  range-frequency distribution. By combining the  $M \times M$  range-frequency distributions at  $N$  Doppler cells, the  $M \times M \times N$  frequency-range-Doppler cube  $Q_2(\omega_m, R_m, f_n)$  can be formed:

$$Q_2(\omega_m, R_m, f_n) = TF_m\{I(R_m, f_n)\}, \quad (6)$$

where  $TF_m$  denotes the time-frequency transform with respect to the variable  $m$ . At each sampling frequency  $\omega_i (i=0, 1, 2, \dots, M-1)$ , only one range-Doppler image frame  $Q_2(\omega_m = \omega_i, R_m, f_n)$  can be extracted from the  $M \times M \times N$  frequency-range-Doppler cube.

Figure 3 shows the typical block diagram of the frequency-range-Doppler processing for generating a 3-D feature space. To retrieve the range-Doppler information, the 2-D FFT is applied to the complex I/Q data array to generate a 2-D range-Doppler image. By taking the time-frequency transform at each Doppler frequency cell along the range domain, the frequency-range-Doppler feature space is generated.

## 5 DETECTION OF MOVING TARGETS IN SAR SCENE

Traditional SAR processing cannot simultaneously produce clear images of stationary targets and moving targets. Moving targets appear as defocused and spatially displaced objects superimposed on the SAR scene [3,4]. In these cases, an important issue is the ability to detect and focus images of moving targets.

Given a radar velocity  $v$  and an initial range from the radar to a moving target  $R_0$ , the Doppler rate of the moving target is determined not only by its geometric location  $(x_0, y_0)$  but also by its velocity and acceleration. If the Doppler rate cannot be compensated in the data, then the image of the moving target becomes defocused.

When the SAR platform is moving along the azimuth direction at an altitude, and the target is moving with a velocity  $v_y$  and an acceleration  $a_y$  in the radial direction, and a velocity  $v_x$  and an acceleration  $a_x$  in the azimuth direction, then the Doppler shift of the returned signal consists of two parts: the part due to the radar motion

$$f_{D_{\text{Radar}}} = -\frac{2}{\lambda} \frac{x_0 v}{R_0} + \frac{2}{\lambda} \frac{v^2}{R_0} t \quad (7)$$

and the part due to the target motion

$$f_{D_{\text{Target}}} = -\frac{2}{\lambda} \frac{x_0 v_x + y_0 v_y}{R_0} + \frac{2}{\lambda} \frac{v_x^2 + v_y^2 + x_0 a_x + y_0 a_y - 2v v_x}{R_0} t \quad (8)$$

where the first term is the Doppler centroid and the second term is the Doppler rate induced by target motion.

The quadratic phase variation between the target and the radar causes de-focusing of the moving target's image. When stationary objects are well focused, the image of moving targets become de-focused and shifted in the cross-range direction.

In order to detect moving targets, estimate the targets' velocities, and relocate mis-located moving targets, multiple-antenna (such as interferometry, planar apertures, or antenna array) approaches are used. Ground moving target indicator (GMTI) using multiple- antenna is used to reject radar returns from clutter and to detect moving target.

The 3-D slow time-range-Doppler feature space provides moving target features. These features include the target's time-varying Doppler spectrum in the 2-D slow time-Doppler slices and targets' motion trajectories in the 2-D range-Doppler slices. Therefore, for the conventional single channel SAR imaging, moving targets can be detected using these features. In particular a chirp response, indicated by a sloped line in the slow time-Doppler domain, is an indication of a moving target in the scene.

Figure 4 shows a spotlight SAR image with a moving vehicle as indicated. AN/APY-6 radar [5] was used to collect the spotlight SAR data. Because of the target motion, the image of the moving vehicle becomes a smeared strip line in the Doppler (cross-range) domain. There are several other strip lines in the image which may not necessarily be moving targets. To verify that the above indicated strip line is a moving target, let us start with the complex I/Q data and perform the slow time-range-Doppler processing.

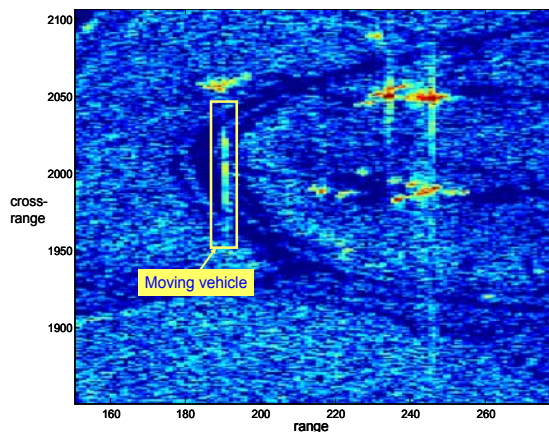


Figure 4. SAR image with a moving vehicle.

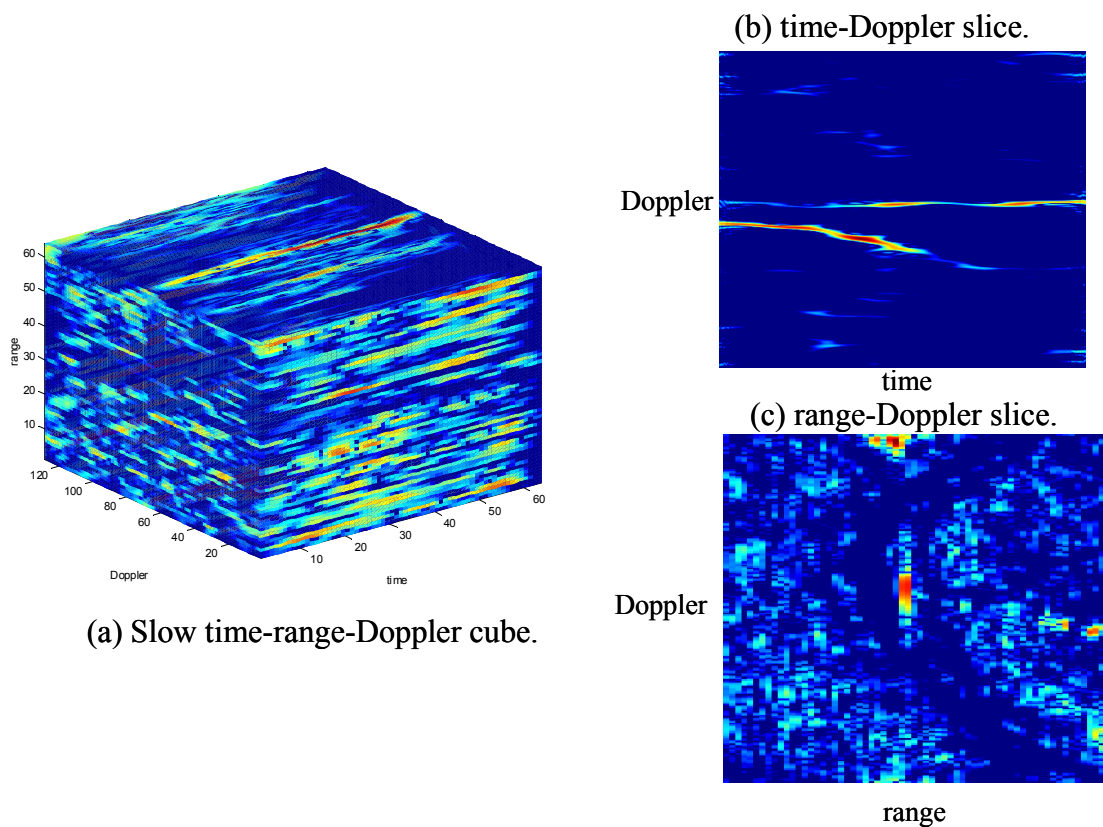


Figure 5. 3-D slow time-range-Doppler processing for moving target detection.



Figure 5 (a) shows the 3-D slow time-range-Doppler cube, (b) shows a 2-D time-Doppler slice which has a sloped time-varying spectrum caused by target motion, and (c) shows a 2-D range-Doppler slice which illustrates the response of the vehicle moving along the cross-range direction. Figure 6 shows the re-focused image of the moving vehicle compared with the unfocused image.

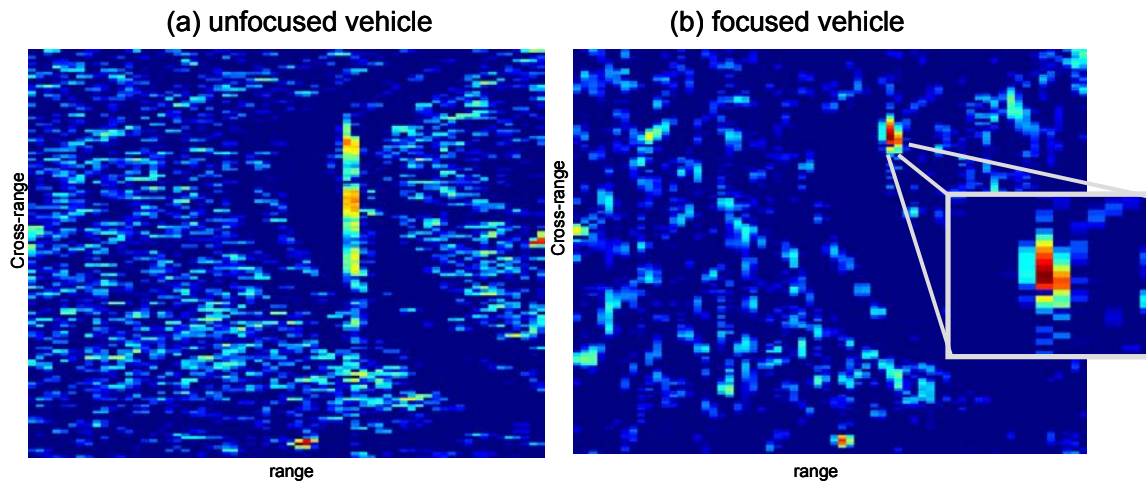


Figure 6. (a) The unfocused image of the moving vehicle; (b) the re-focused image of the moving vehicle by using the slow time-range-Doppler processing.

## 6 SUMMARY

Using the joint time-frequency transform, a time series can be transformed into a 2-D joint time and frequency space, and a 2-D complex I/Q radar data can be transformed into a 3-D feature space. The 3-D feature spaces can be derived from various combinations of time-frequency transform applied to the fast time and the slow time domains. In this paper, we discussed the 3-D slow time-range-Doppler space and its application to moving target detection and re-focusing. A SAR scene with a moving vehicle generated from AN/APY-6 radar data is used to demonstrate the usefulness of the slow time-range-Doppler feature space for detecting and focusing ground moving targets.

## 7 ACKNOWLEDGEMENT

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